BID: Kinetic Energy (average speed) of molecules

The Big Idea

The kinetic energy of an ideal gas (and/or its molecules) is directly proportional to the absolute temperature of the gas. ($K_{avg} = {}^{3}/_{2}\, k_B T$) From this, average (rms) speed can be found. ($\upsilon_{rms} = \sqrt{\frac{3k_B T}{\mu}}$)

More details

Boltzmann's constant (**k_B**) is simply the Universal Gas Constant (**R**) divided by Avogadro's number. It sort of acts as "R" for single molecules. If the problem involves a mole (or moles) of gas rather than molecules, simply use R instead of Boltzmann's constant.

How to recognize it

You will be given the temperature of a gas (in °C or K). For a speed of molecules problem, you'll also be told what type of molecule, so you can determine the molecular mass. The problem will either ask you for the average kinetic energy of a gas or the rms average speed of the molecules in a gas.

How to tackle it

Basically just "plug and chug" using the two formulae listed above.

Pitfalls to watch for

1) Temperature must be in kelvins!!
2) For molecular mass, you need to find the mass in amu and then multiply that by $1.67\text{x}10^{-27}$ kg.
3) When finding the average speed of the molecules for a mixed gas, you need to find the speeds separately for each type of molecule.

Example problems

1) *What is the average kinetic energy of molecules in a gas at 75 °C?*

Solution:

$$\begin{aligned} K_{avg} &= {}^{3}/_{2}\, k_B T \\ &= 1.5 \cdot 1.38\text{x}10^{-23}\ J/K \cdot 348\ K \\ &= 7.2036\text{x}10^{-21}\ J \\ &= \boxed{7.20\text{x}10^{-21}\ J} \end{aligned}$$

2) *What is the average molecular speed of carbon dioxide molecules at 37 °C?*

Solution:

$$\begin{aligned} \upsilon_{rms} &= \sqrt{\frac{3k_B T}{\mu}} \\ &= \sqrt{\frac{3 \cdot 1.38x10^{-23}\ J/K \cdot 310K}{44 \cdot 1.67x10^{-27}\ kg}} \\ &= \sqrt{174659.77} \\ &= \boxed{417.9\ \text{m/s}} \end{aligned}$$